Solving Probabilistic Inverse Problems Using Factor Graph Analysis

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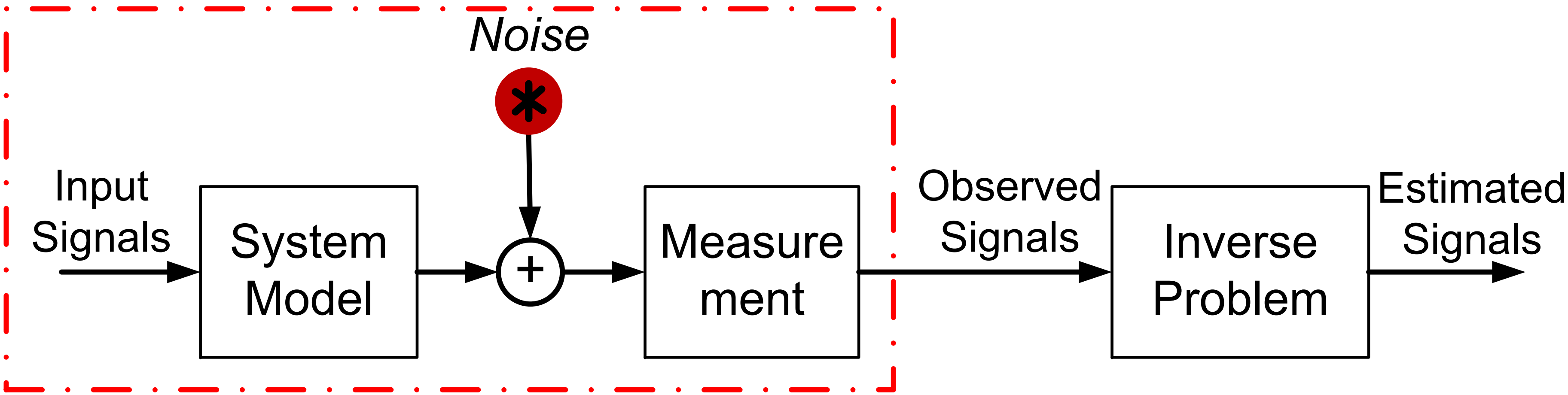
Outline

- What is inverse problem?
- Inverse problem examples
- Solving inverse problems
  - Factor graph
  - Message passing algorithm
- Analyzing convergence of message passing algorithm with ***density evolution*** (DE)
- Simulation results
  - For communication
  - For bio-sensing
- Conclusion

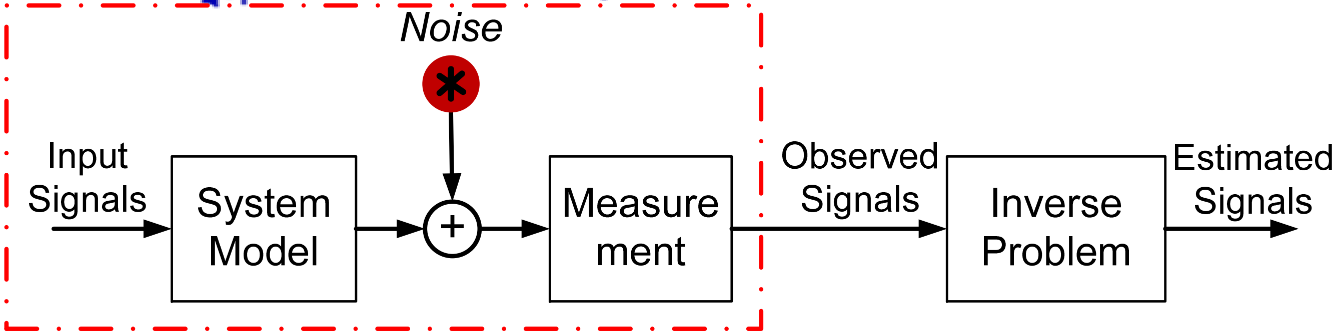
What is inverse problem?

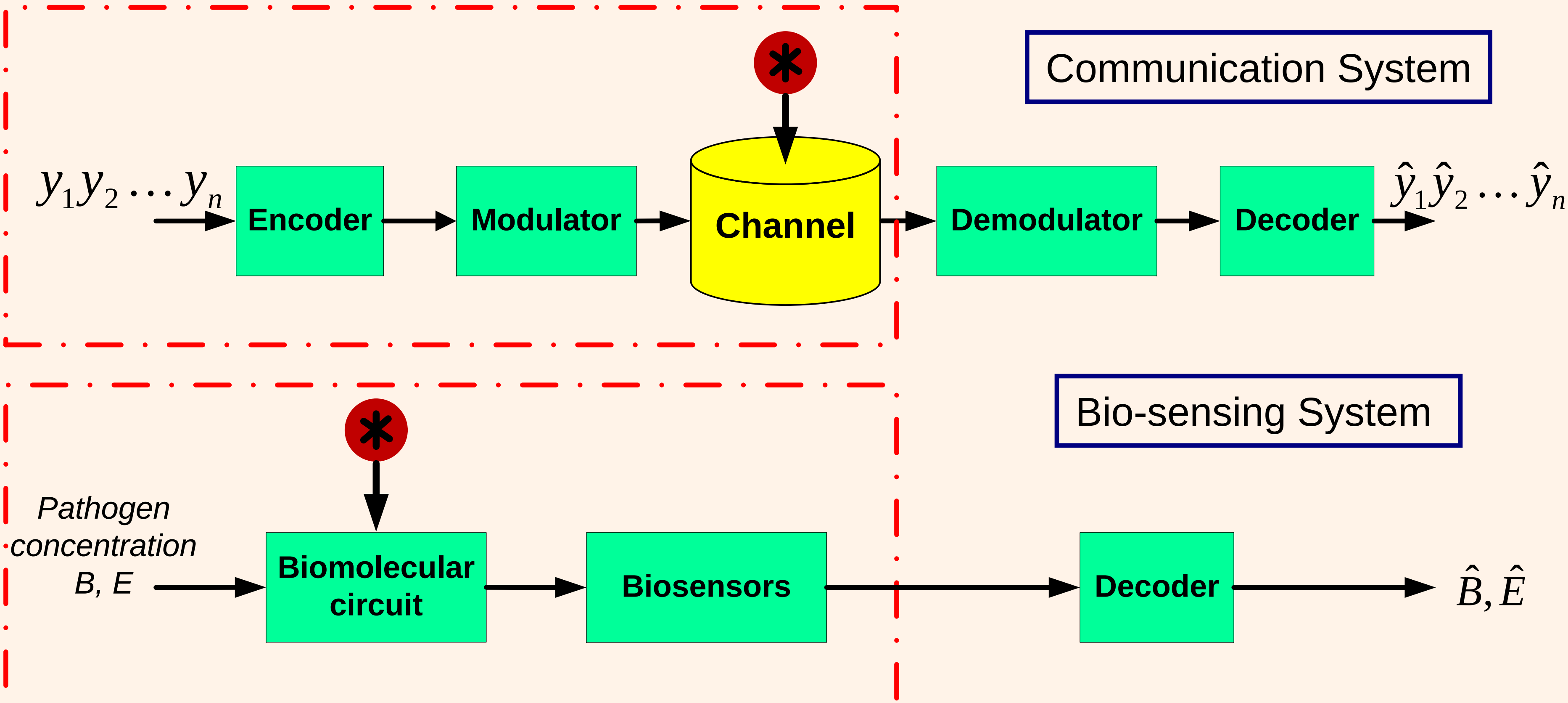
The *inverse problem* consists of using the actual result of some measurements to infer the values of the parameters that characterize the system.

--Albert Tarantola, “Inverse Problem Theory and Methods for Model Parameter Estimation”, Society for Industrial and Applied Mathematics, Dec. 2004.

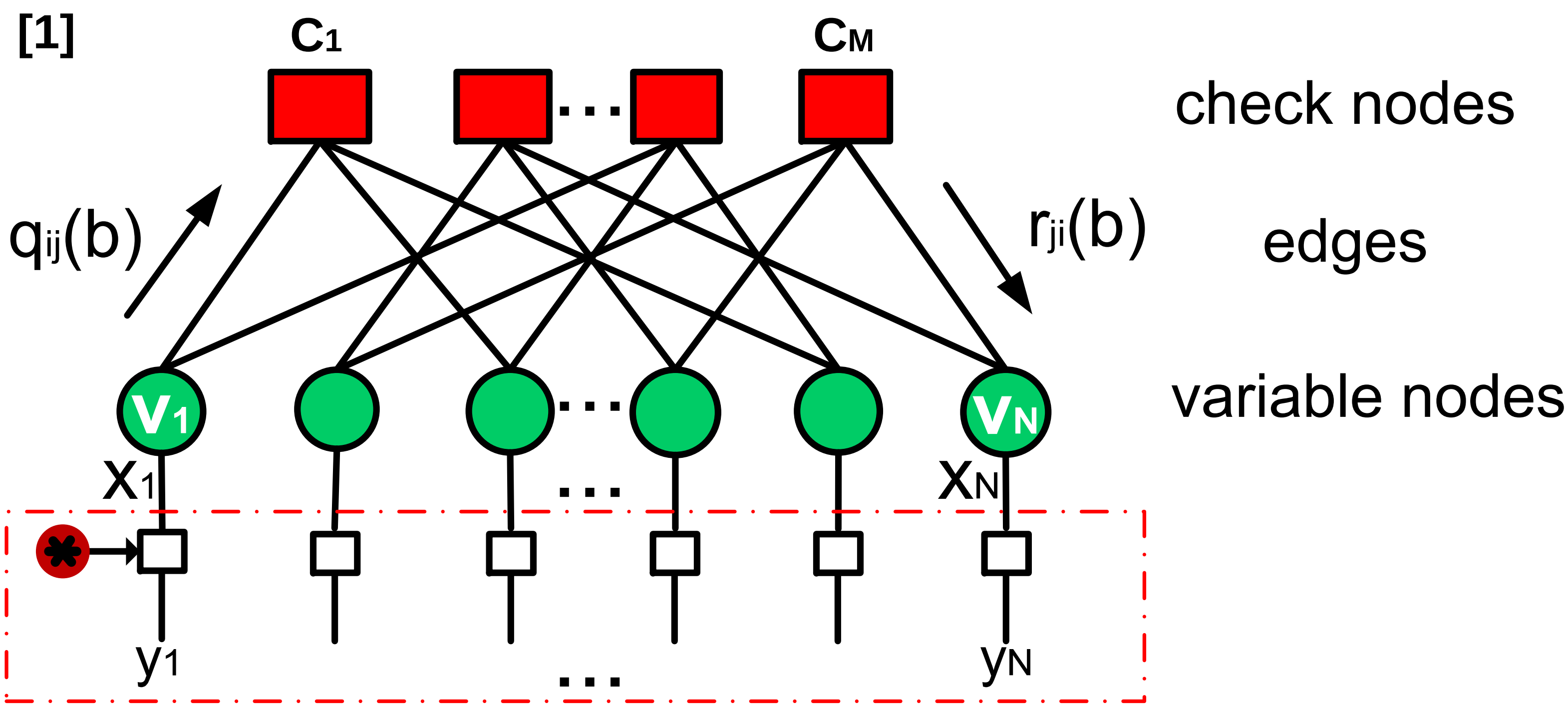


Inverse problem





Factor graph and message passing algorithm

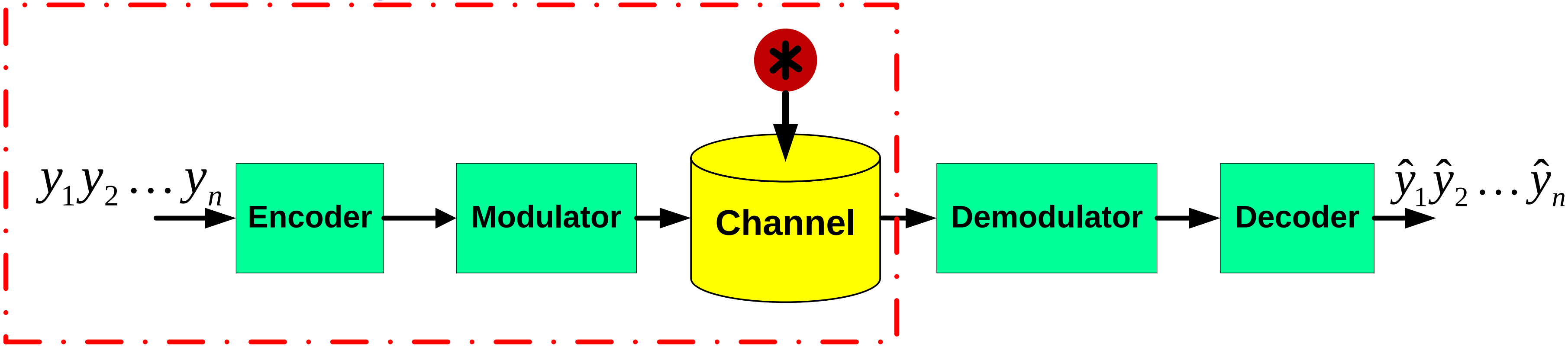


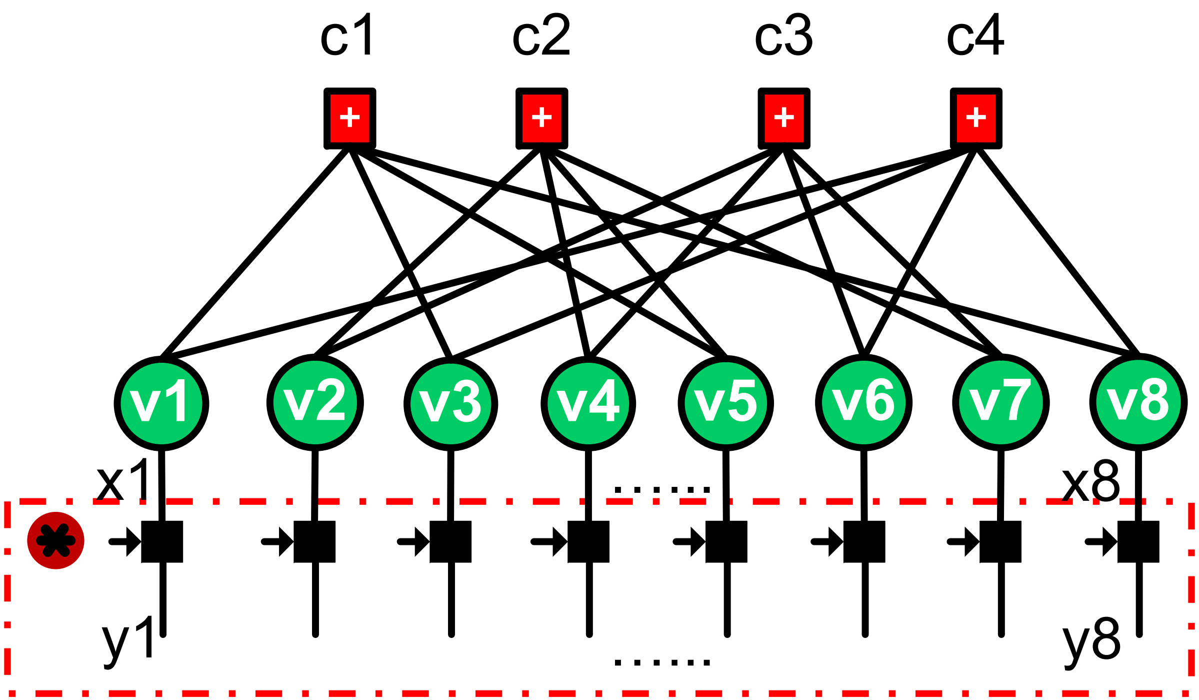
$$b \in Value\, domain\, of\ y_i, i \in [1, N]$$

qij(b): $P(v_i = b \mid x_i\ received, check\ equations\ involving\ v_i\ are\ satisfied), i \in [1, N]$

rji(b): $P(check\ equation\ c_j\ is\ satisfied \mid v_i = b), i \in [1, N], j \in [1, M]$

Factor graph in communication



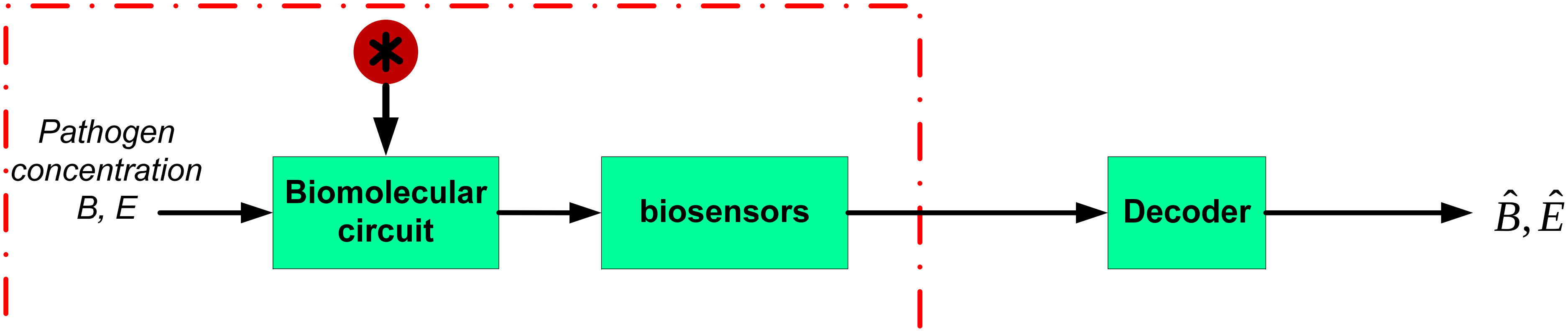


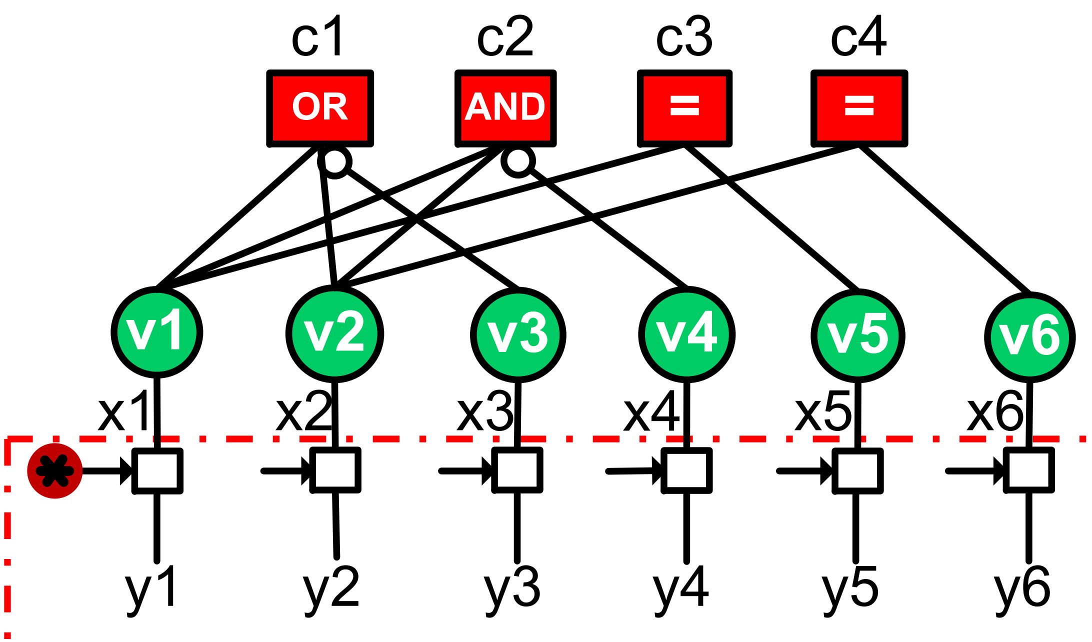
$$\hat{y} = v = ?$$

$$Hv = 0$$

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Factor graph in bio-sensing

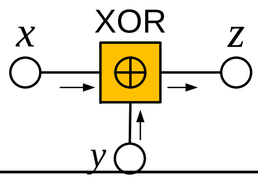
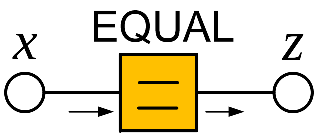
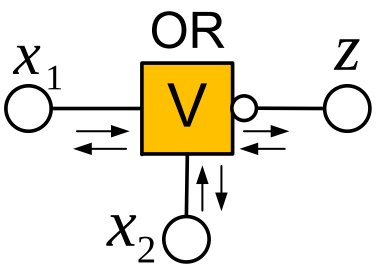
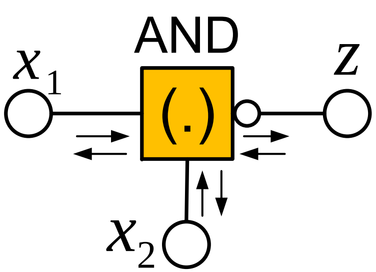




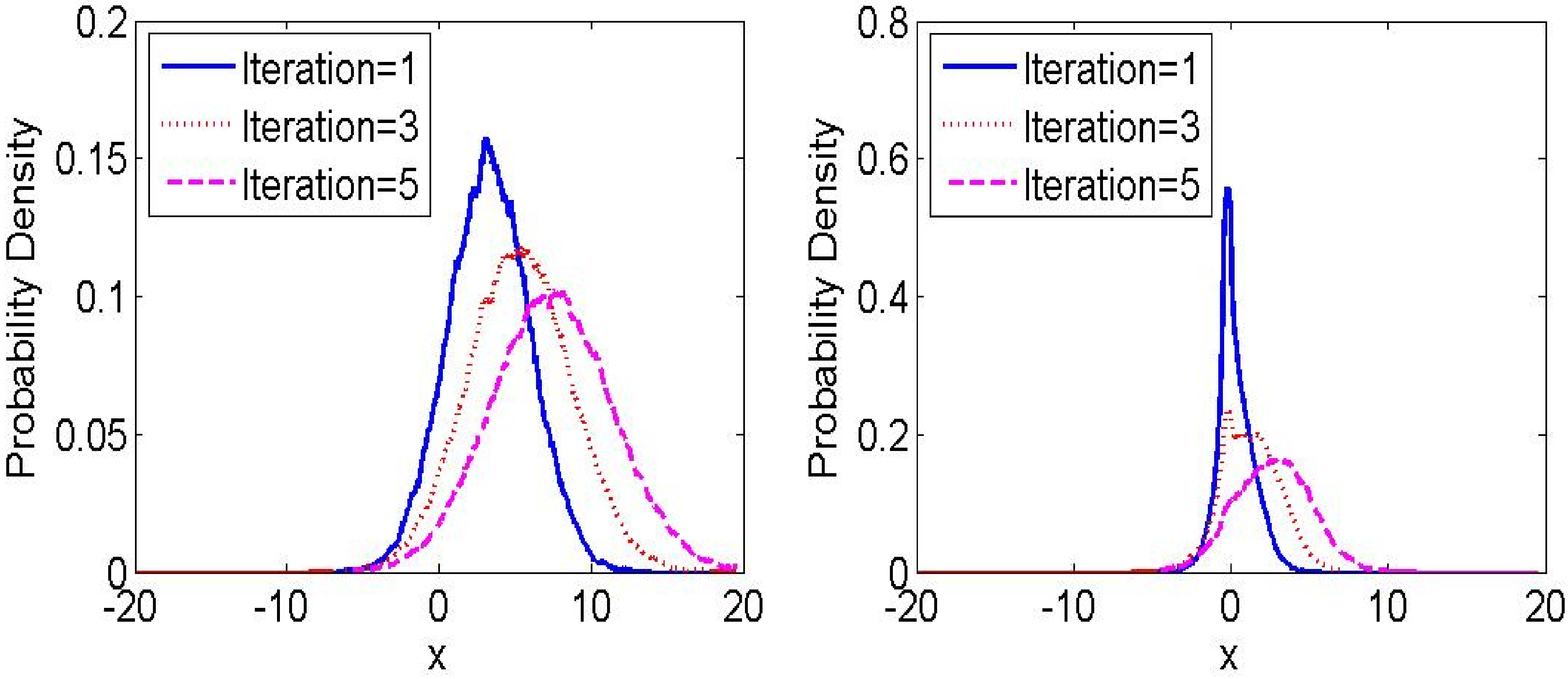
$$\hat{y} = v = \text{?}$$

$$c_1: \quad v_1 \otimes v_2 = v_3$$
$$c_2: \quad v_1 \cdot v_2 = v_4$$
$$c_3: \quad v_5 = v_1$$
$$c_4: \quad v_6 = v_2$$

Message passing algorithm

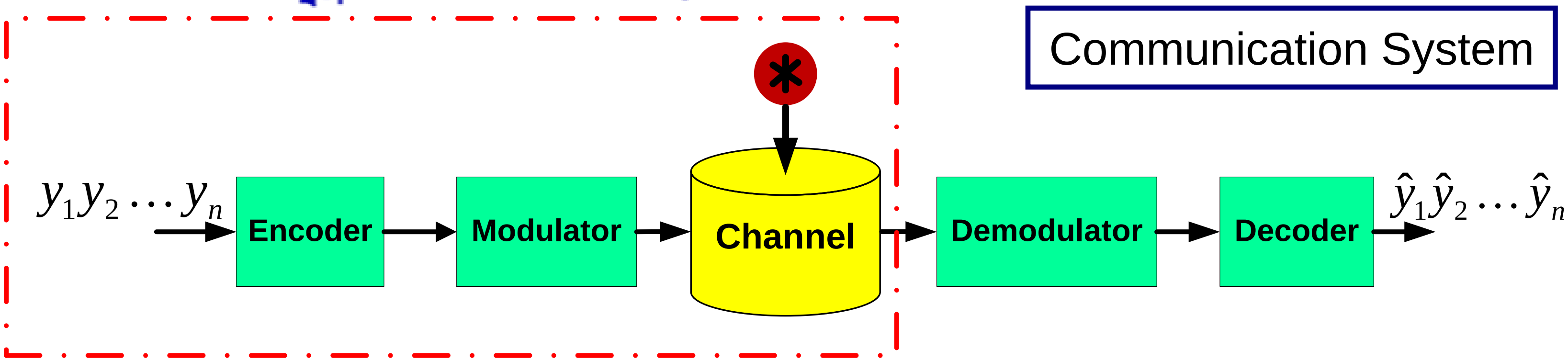
	$\begin{pmatrix}\mu_z(0)\\ \mu_z(1)\end{pmatrix} = \begin{pmatrix}\mu_x(0)\mu_y(0)+\mu_x(1)\mu_y(1)\\ \mu_x(0)\mu_y(1)+\mu_x(1)\mu_y(0)\end{pmatrix}$
	$\begin{pmatrix}\mu_z(0)\\ \mu_z(1)\end{pmatrix} = \begin{pmatrix}\mu_x(0)\\ \mu_x(1)\end{pmatrix}$
	$\begin{pmatrix}\mu_{OR\to x_1}(0)=\mu_{x_2\to OR}(0)\mu_{z\to OR}(0)+\mu_{x_2\to OR}(1)\mu_{z\to OR}(1)\\ \mu_{OR\to x_1}(1)=\mu_{x_2\to OR}(0)\mu_{z\to OR}(1)+\mu_{x_2\to OR}(1)\mu_{z\to OR}(1)\end{pmatrix}$ $\begin{pmatrix}\mu_{OR\to x_2}(0)=\mu_{x_1\to OR}(0)\mu_{z\to OR}(0)+\mu_{x_1\to OR}(1)\mu_{z\to OR}(1)\\ \mu_{OR\to x_2}(0)=\mu_{x_1\to OR}(0)\mu_{z\to OR}(1)+\mu_{x_1\to OR}(1)\mu_{z\to OR}(1)\end{pmatrix}$ $\begin{pmatrix}\mu_{OR\to z}(0)=\mu_{x_1\to OR}(0)\mu_{x_2\to OR}(0)\\ \mu_{OR\to z}(1)=\mu_{x_1\to OR}(0)\mu_{x_2\to OR}(1)+\mu_{x_1\to OR}(1)\mu_{x_2\to OR}(0)+\mu_{OR\to z}(0)=\mu_{x_1\to OR}(0)\mu_{x_2\to OR}(0)\end{pmatrix}$
	$\begin{pmatrix}\mu_{AND\to x_1}(0)=\mu_{x_2\to AND}(0)\mu_{z\to AND}(0)+\mu_{x_2\to AND}(1)\mu_{z\to AND}(0)\\ \mu_{AND\to x_1}(1)=\mu_{x_2\to AND}(0)\mu_{z\to AND}(0)+\mu_{x_2\to AND}(1)\mu_{z\to AND}(1)\end{pmatrix}$ $\begin{pmatrix}\mu_{AND\to x_2}(0)=\mu_{x_1\to AND}(0)\mu_{z\to AND}(0)+\mu_{x_1\to AND}(1)\mu_{z\to AND}(0)\\ \mu_{AND\to x_2}(1)=\mu_{x_1\to AND}(0)\mu_{z\to AND}(0)+\mu_{x_1\to AND}(1)\mu_{z\to AND}(1)\end{pmatrix}$ $\begin{pmatrix}\mu_{AND\to z}(0)=\mu_{x_1\to AND}(0)\mu_{x_2\to AND}(0)+\mu_{x_1\to AND}(0)\mu_{x_2\to AND}(1)+\mu_{x_1\to AND}(1)\mu_{x_2\to AND}(0)\\ \mu_{AND\to z}(1)=\mu_{x_1\to AND}(1)\mu_{x_2\to AND}(1)\end{pmatrix}$

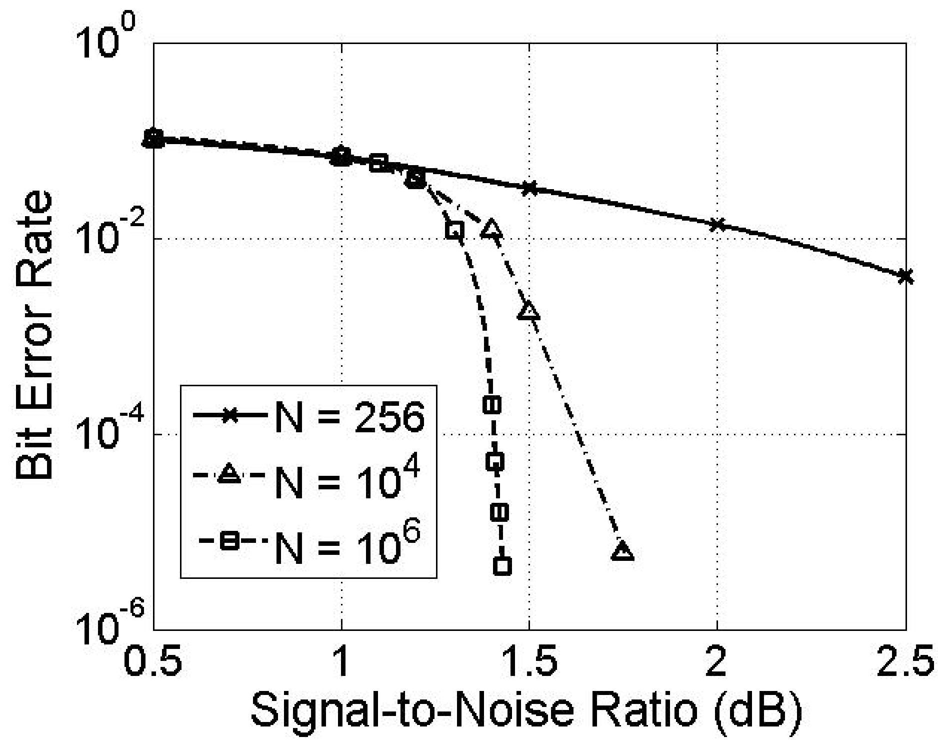
Density evolution



-- Thomas J. Richardson, M. Amin Shokrollahi, Rüdiger L. Urbanke, “Design of Capacity-Approaching Irregular Low-density Parity-Check Codes”, IEEE Transactions on information theory, Vol. 47, No. 2, Feb. 2001.

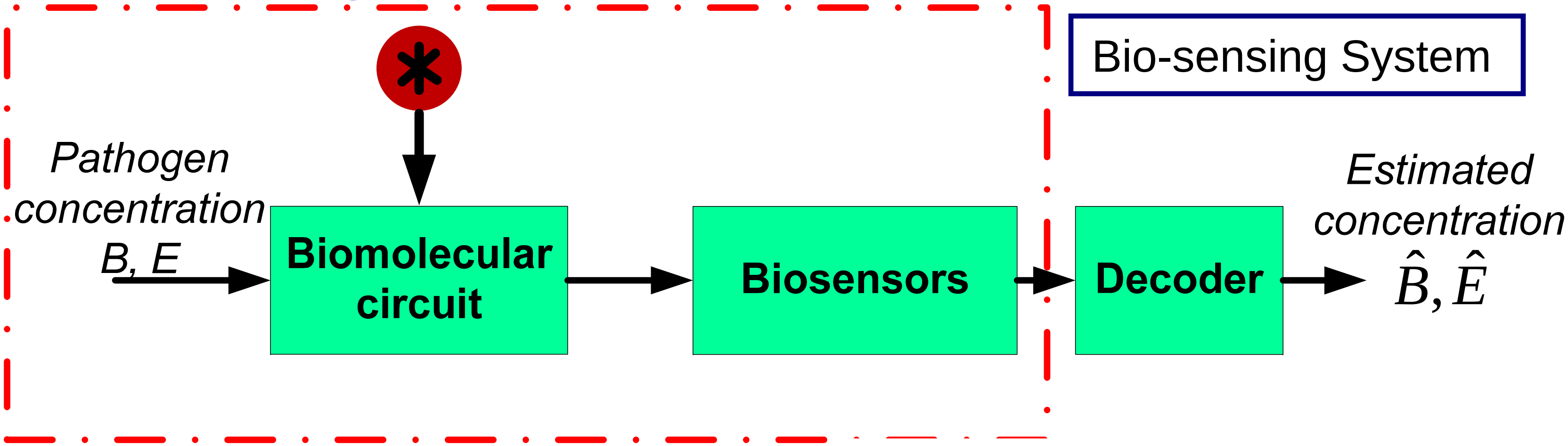
Simulation Results



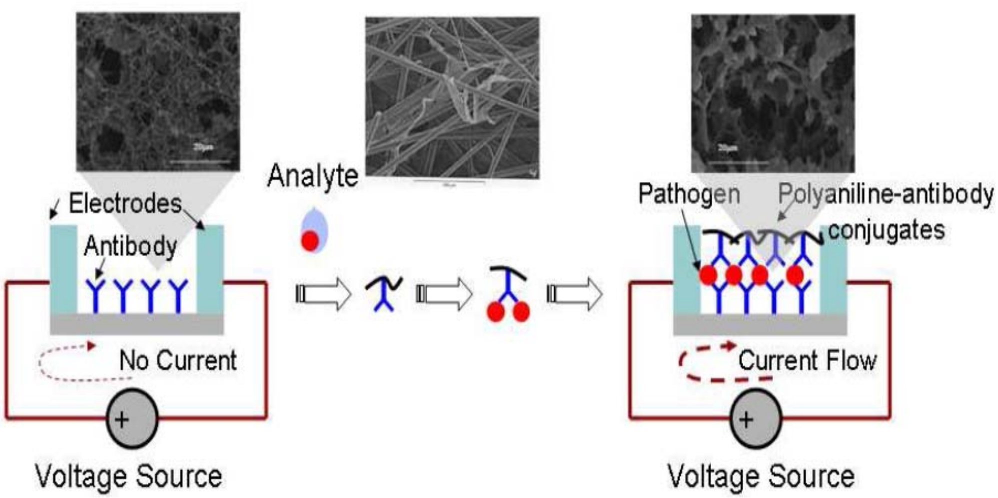


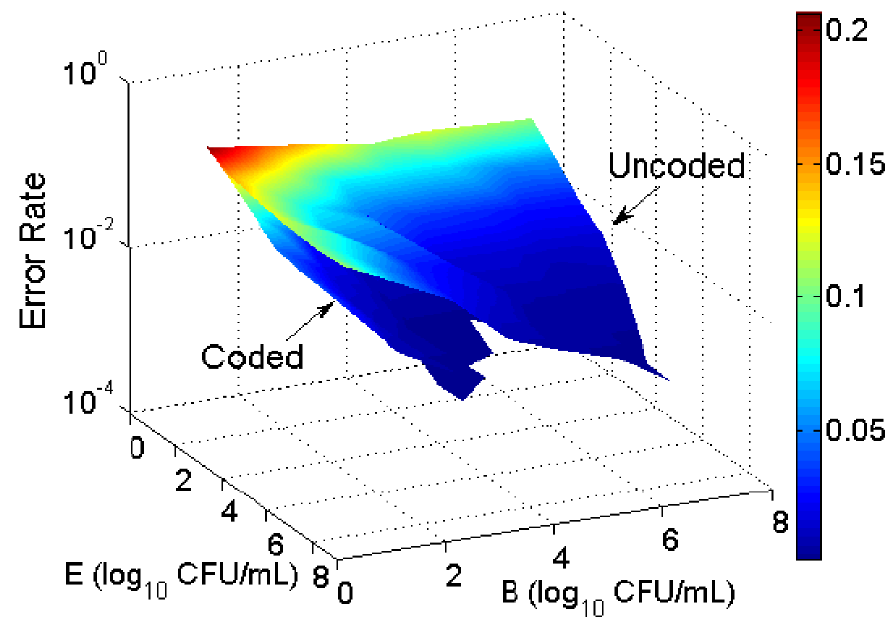
***C++ open source code will be available online soon.**

Simulation Results



[2]





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Conclusion

- Combined with iterative message passing algorithm, factor graph can serve as an important analysis and visualization tool that can produce probabilistic estimates of the unknown model parameters.
- Factor-graph based inverse analysis can be used for reliable communications and for biomolecular circuit analysis.

References

[1] H.-A. Loeliger, “An introduction to factor graphs,” IEEE Signal Proc. Mag., pp. 28-41, Jan. 2004.

[2] Y. Liu, S. Chakrabartty, “Factor graph based biomolecular circuit analysis for designing forward error correcting biosensors”, IEEE Transactions of Biomedical Circuits and Systems (To appear 2009).

Thank you!